CALCULATION OF ENTROPY FROM ACOUSTIC DATA

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In [1] Novikov and Trelin proposed a graphoanalytical method of calculating entropy diagrams from experimental data on the speed of sound and the p, V, T relation. This method is based on two well-known thermodynamic equations for an isentropic process:

$$\left(\frac{\partial V}{\partial p}\right)_S = -\frac{V^2}{a^2}, \qquad \Delta V = \int_{p_1}^{p_2} \left(\frac{\partial V}{\partial p}\right)_S dp. \tag{1}$$

Here, a is the speed of sound, and ΔV is the change in specific volume in the isentropic process when the pressure changes from p_1 to P_2.

It is clear from (1) that in such a calculation differentiation of the experimental data is eliminated. The variation of the quantity $(\partial V/\partial p)_S$ along the isentrope as a function of pressure is not known in advance; therefore it has been proposed to evaluate the integral in the expression for ΔV by successive approximations using two auxiliary graphs: the isobar in V, T and $(\partial V/\partial p)_S$ coordinates. The details of the calculation are described in [1].

Below, an attempt is made to free this method of graphical constructions.

For each isobar the exerpimental data on the p, V, T relation can be represented in the form $V = f(T)$.

Similarly, for each isobar we can write the temperature dependence of the derivative $(\partial V/\partial p)_S$ calculated from (1):

$$(\partial V/\partial p)_S = \varphi(T). \tag{2}$$

The integral in the right side of the second of expressions (1) can be evaluated for any isentrope using one of the approximate integration formulas, e.g., the trapezoidal rule,

$$\int_{p_1}^{p_2} \left(\frac{\partial V}{\partial p}\right)_S dp = \frac{1}{2}\left[\varphi(T) + \left(\frac{\partial V}{\partial p}\right)_{S1}\right](p_2 - p_1). \tag{3}$$

Here, $(\partial V/\partial p)_{S1}$ is the value of the derivative at pressure p_1, i.e., for the first point of the isentrope, and $\varphi(T)$ is the still unknown value of the derivative at pressure p_2, which depends on the temperature at the point of intersection of the given isentrope and the p_2 isobar. This same temperature uniquely determines the value of the specific volume at the point of intersection of the isentrope and the p_2 isobar; therefore, substituting $V = f(T)$ and (3) in the second of relations (1), we obtain an equation for the unknown temperature

$$f(T) - V_1 = \frac{1}{2}\left[\varphi(T) + \left(\frac{\partial V}{\partial p}\right)_{S1}\right](p_2 - p_1) \tag{4}$$

or

$$F(T) + A = 0. \tag{5}$$

Here, A is the free term of the equation, and V_1 is the value of the specific volume for the first point of the isentrope, i.e., at pressure p_1.

The above reasoning is equally valid for any other isentrope; consequently, having solved Eq. (5), we can obtain a single formula from which we then determine the temperature for the points of intersection of different isentropes with the p_2 isobar.

The functions $f(T)$, $\varphi(T)$, and hence F(T), usually take the form of polynomials, it often being possible to confine oneself to polynomials of the second or third dgreee. For a polynomial of the second degree the solution of Eq. (5) has the following form:

$$T = a - \sqrt{b + cx}. \tag{6}$$

Here, a, b, c are constants which are evaluated by performing simple arithmetic operations on the coefficients of the polynomials $f(T)$ and $\varphi(T)$, and the parameter x is expressed in terms of the quantities $(\partial V/\partial p)_{S1}$ and V_1, i.e., it has different values for different isentropes.

Thus, by substituting different values of x in (6), we can calculate values of the temperature along the isobar for all the given isentropes. The corresponding values of V and $(\partial V/\partial p)_S$ are calculated from the relations $V = f(T)$ and (2) and are then used to evaluate the parameter x for the next isobar. When calculated values of V and $(\partial V/\partial p)_S$ are available for at least two isobars it is desirable to use the more accurate Simpson's rule as an integration formula. In this case, instead of (3) we obtain

$$\int_{p_1}^{p_3} \left(\frac{\partial V}{\partial p}\right)_S dp = \frac{1}{6}\left[\varphi(T) + \left(\frac{\partial V}{\partial p}\right)_{S1} + 4\left(\frac{\partial V}{\partial p}\right)_{S2}\right](p_3 - p_1). \tag{7}$$

As initial data it is possible to use the entropy data on the saturation line.

Entropy of Argon (J/kg • °K)

T, °K	$p = 10$	20	30	40	50	60	70	75
120	2870							
130	2938							
140		2780	2614					
150		2836	2704	2570				
160		2887	2764	2660	2557			
170		2935	2813	2723	2635	2556	2482	2450
180			2856	2769	2697	2624	2562	2534
190			2898	2813	2743	2681	2624	2598
200			2937	2852	2786	2728	2676	2649
210				2889	2822	2769	2719	2696
220				2925	2857	2805	2758	2735
230					2891	2837	2793	2772
240					2923	2868	2825	2805
250					2952	2898	2854	2835
260						2927	2882	2863
270						2955	2910	2889

The proposed method was used to calculate the entropy of argon at 120–270°K and 10–75 bar. In these calculations we used data on the speed of sound in argon from [2–4] and on the density of argon from [5]. Values of the entropy on the saturation curve were taken from [6]. The results are presented in the table.

The changes in entropy along the 40 and 70-bar isobars were compared with the corresponding values based on Michel's data [7]. The discrepancy is of the order of 0.5–1%.

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